SINP/TNP/02-26 hep-ph/0209053

Inverse beta-decay of arbitrarily polarized neutrons in a magnetic field

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Abstract

We calculate the cross section of the inverse beta decay process, $\nu_e + n \to p + e$, in a background magnetic field which is much smaller than m_p^2/e. Using exact solutions of the Dirac equation in a constant magnetic field, we find the cross section for arbitrary polarization of the initial neutrons. The cross section depends on the direction of the incident neutrino even when the initial neutron is assumed to be at rest and has no net polarization. Possible implications of the result are discussed.

1 Introduction

The interactions of elementary particles show novel features when they occur in non-trivial backgrounds. Study of particle propagation in matter has proved pivotal in the understanding of the solar neutrino problem. Similar studies of particle processes in background magnetic fields are also important since stellar objects like neutron stars are expected to possess very high magnetic fields, of the order of 10^{12} G or higher. Analysis of these processes might be crucial for obtaining a proper understanding of the properties of these stars.

In this paper, we calculate the cross section of the inverse beta-decay process in a magnetic field. We consider the possibility that the neutrons may be totally or partially polarized in the magnetic field, and find the cross section as a function of this polarization. The neutrinos are assumed to be strictly standard model neutrinos, without any mass and consequent properties. The presence of the magnetic field breaks the isotropy of the background, and a careful calculation in this background reveals a dependence of the cross section on the incident neutrino direction with respect to the magnetic field.

Considerable work has been done on the magnetic field dependence of the URCA processes which have neutrinos in their final states [1, 2, 3, 4, 5]. An angular dependence obtained in the differential cross section of these reactions imply that in a star with high magnetic field, neutrinos are created asymmetrically with respect to the magnetic field direction. The process that we consider, on the other hand, have neutrinos in the initial state. So this process influences the neutrino opacity in a star.

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Some calculations of this process exist in the literature. Roulet [6], as well as Lai and Qian [7] performed the calculation by assuming that the magnetic field effects enter only through the phase space integrals, whereas the matrix element remains unaffected. Gvozdev and Ognev [3] considered the final electron to be exclusively in the lowest Landau level. Arras and Lai [4] calculated only the angular asymmetry, and only to the first order in the background magnetic field. The earlier calculations of the present authors [8, 9] did not take neutron polarization into account. In this paper, we consider the problem in full detail — i.e., we calculate the matrix element using spinor solutions of the electron in a magnetic field, take all possible final Landau levels into account, include the possiblity of neutron polarization, and perform the calculations to all orders in the background field in the 4-fermi interaction theory.

The paper is organized as follows. In Sec 2, we provide some background for the calculation. Most of this section contains no original material, but we provide it for the sake of completeness, as well as for setting up the notation that will be used in the later sections. In Sec. 3, we define the fermion field operator and show how it acts on the states in the presence of a magnetic field. Sec. 4 contains the calculation of the cross section for a monochromatic neutrino beam, which contains the main results of our paper. In Sec. 5, we discuss the realistic case where the initial neutrino beam has a finite energy spread. Sec. 6 contains our discussions and conclusions.

2 Solutions of the Dirac equation in a uniform magnetic field

For a particle of charge eQ, the Dirac equation in presence of a magnetic field is given by

$$i\frac{\partial\psi}{\partial t} = [\vec{\alpha}\cdot(-i\vec{\nabla} - eQ\vec{A}) + \beta m]\psi \,, \tag{2.1}$$

where $\vec{\alpha}$ and β are the Dirac matrices, and $\vec{A}$ is the vector potential. In our convention, e is the positive unit of charge, taken as usual to be equal to the proton charge.

For stationary states, we can write

$$\psi = e^{-iEt}\begin{pmatrix}\phi\\ \chi\end{pmatrix} , \tag{2.2}$$

where ϕ and χ are 2-component objects. We use the Pauli-Dirac representation of the Dirac matrices, in which

$$\vec{\alpha} = \begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix} , \qquad \beta = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{2.3}$$

where each block represents a 2×2 matrix, and $\vec{\sigma}$ are the Pauli matrices. With this notation, we can write Eq. (2.1) as

$$(E-m)\phi = \vec{\sigma}\cdot(-i\vec{\nabla} - eQ\vec{A})\chi \,, \tag{2.4}$$

$$(E+m)\chi = \vec{\sigma}\cdot(-i\vec{\nabla} - eQ\vec{A})\phi \,. \tag{2.5}$$

Eliminating χ, we obtain

$$(E^2-m^2)\phi = \Big[\vec{\sigma}\cdot(-i\vec{\nabla} - eQ\vec{A})\Big]^2\phi \,. \tag{2.6}$$

We will work with a constant magnetic field $\vec{B}$. Without loss of generality, it can be taken along the z-direction. The vector potential can be chosen in many equivalent ways. We take

$$A_0 = A_y = A_z = 0\,, \qquad A_x = -yB\,. \tag{2.7}$$

With this choice, Eq. (2.6) reduces to the form

$$(E^2 - m^2)\phi = \Big[-\vec{\nabla}^2 + (eQB)^2 y^2 - eQB(2iy\frac{\partial}{\partial x} + \sigma_z)\Big]\phi\,. \tag{2.8}$$

Noticing that the co-ordinates x and z do not appear in the equation except through the derivatives, we can write the solutions as

$$\phi = e^{i\vec{p}\cdot\vec{X}_{\not y}} f(y)\,, \tag{2.9}$$

where $f(y)$ is a 2-component matrix which depends only on the y-coordinate, and possibly some momentum components, as we will see shortly. We have also introduced the notation $\vec{X}$ for the spatial co-ordinates (in order to distinguish it from x, which is one of the components of $\vec{X}$), and $\vec{X}_{\not y}$ for the vector $\vec{X}$ with its y-component set equal to zero. In other words, $\vec{p}\cdot\vec{X}_{\not y} \equiv p_x x + p_z z$, where p_x and p_z denote the eigenvalues of momentum in the x and z directions.[1]

There will be two independent solutions for $f(y)$, which can be taken, without any loss of generality, to be the eigenstates of σ_z with eigenvalues $s = \pm 1$. This means that we choose the two independent solutions in the form

$$f_+(y) = \begin{pmatrix} F_+(y) \\ 0 \end{pmatrix}, \qquad f_-(y) = \begin{pmatrix} 0 \\ F_-(y) \end{pmatrix}. \tag{2.10}$$

Since $\sigma_z F_s = sF_s$, the differential equations satisfied by F_s is

$$\frac{d^2 F_s}{dy^2} - (eQBy + p_x)^2 F_s + (E^2 - m^2 - p_z^2 + eQBs)F_s = 0\,, \tag{2.11}$$

which is obtained from Eq. (2.8). The solution is obtained by using the dimensionless variable

$$\xi = \sqrt{e|Q|B}\left(y + \frac{p_x}{eQB}\right), \tag{2.12}$$

which transforms Eq. (2.11) to the form

$$\left[\frac{d^2}{d\xi^2} - \xi^2 + a_s\right] F_s = 0\,, \tag{2.13}$$

where

$$a_s = \frac{E^2 - m^2 - p_z^2 + eQBs}{e|Q|B}\,. \tag{2.14}$$

This is a special form of Hermite's equation, and the solutions exist provided $a_s = 2\nu + 1$ for $\nu = 0, 1, 2, \cdots$. This provides the energy eigenvalues

$$E^2 = m^2 + p_z^2 + (2\nu + 1)e|Q|B - eQBs\,, \tag{2.15}$$

[1] It is to be understood that whenever we write the spatial component of any vector with a lettered subscript, it would imply the corresponding contravariant component of the relevant 4-vector.

and the solutions for F_s are

$$N_\nu e^{-\xi^2/2} H_\nu(\xi) \equiv I_\nu(\xi)\,, \tag{2.16}$$

where H_ν are Hermite polynomials of order ν, and N_ν are normalizations which we take to be

$$N_\nu = \left(\frac{\sqrt{e|Q|B}}{\nu!\, 2^\nu \sqrt{\pi}} \right)^{1/2} . \tag{2.17}$$

We stress that the choice of normalization can be arbitrarily made, as will be clarified later. With our choice, the functions I_ν satisfy the completeness relation

$$\sum_\nu I_\nu(\xi) I_\nu(\xi_\star) = \sqrt{e|Q|B}\; \delta(\xi - \xi_\star) = \delta(y - y_\star)\,, \tag{2.18}$$

where $\xi_\star$ is obtained by replacing y by $y_\star$ in Eq. (2.12).

So far, Q was arbitrary. We now specialize to the case of electrons, for which $Q = -1$. The solutions are then conveniently classified by the energy eigenvalues

$$E_n^2 = m^2 + p_z^2 + 2neB\,, \tag{2.19}$$

which is the relativistic form of Landau energy levels. The solutions are two fold degenerate in general: for $s = 1$, $\nu = n - 1$ and for $s = -1$, $\nu = n$. In the case of $n = 0$, only the second solution is available since ν cannot be negative. The solutions can have positive or negative energies. We will denote the positive square root of the right side by E_n. Representing the solution corresponding to this n-th Landau level by a superscript n, we can then write for the positive energy solutions,

$$f_+^{(n)}(y) = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \end{pmatrix}, \qquad f_-^{(n)}(y) = \begin{pmatrix} 0 \\ I_n(\xi) \end{pmatrix}. \tag{2.20}$$

For $n = 0$, the solution f_+ does not exist. We will consistently incorporate this fact by defining

$$I_{-1}(y) = 0\,, \tag{2.21}$$

in addition to the definition of I_n in Eq. (2.16) for non-negative integers n.

The solutions in Eq. (2.20) determine the upper components of the spinors through Eq. (2.9). The lower components, denoted by χ earlier, can be solved using Eq. (2.5), and finally the positive energy solutions of the Dirac equation can be written as

$$e^{-ip\cdot X_{\not y}} U_s(y, n, \vec{p}_{\not y})\,, \tag{2.22}$$

where X^μ denotes the space-time coordinate. And U_s are given by

$$U_+(y, n, \vec{p}_{\not y}) = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \\ \dfrac{p_z}{E_n + m} I_{n-1}(\xi) \\ -\dfrac{\sqrt{2neB}}{E_n + m} I_n(\xi) \end{pmatrix}, \qquad U_-(y, n, \vec{p}_{\not y}) = \begin{pmatrix} 0 \\ I_n(\xi) \\ -\dfrac{\sqrt{2neB}}{E_n + m} I_{n-1}(\xi) \\ -\dfrac{p_z}{E_n + m} I_n(\xi) \end{pmatrix}. \tag{2.23}$$

A similar procedure can be adopted for negative energy spinors which have energy eigenvalues $E = -E_n$. In this case, it is easier to start with the two lower components first and then find the upper components from Eq. (2.4). The solutions are

$$e^{ip\cdot X_{\bar{y}}} V_s(y, n, \vec{p}_{\bar{y}}) \,, \tag{2.24}$$

where

$$V_+(y, n, \vec{p}_{\bar{y}}) = \begin{pmatrix} \dfrac{p_z}{E_n + m} I_{n-1}(\widetilde{\xi}) \\ \dfrac{\sqrt{2neB}}{E_n + m} I_n(\widetilde{\xi}) \\ I_{n-1}(\widetilde{\xi}) \\ 0 \end{pmatrix} , \qquad V_-(y, n, \vec{p}_{\bar{y}}) = \begin{pmatrix} \dfrac{\sqrt{2neB}}{E_n + m} I_{n-1}(\widetilde{\xi}) \\ -\dfrac{p_z}{E_n + m} I_n(\widetilde{\xi}) \\ 0 \\ I_n(\widetilde{\xi}) \end{pmatrix} . \tag{2.25}$$

where $\widetilde{\xi}$ is obtained from ξ by changing the sign of the p_x-term.

For future use, we note down a few identities involving the spinors which can be obtained by direct substitutions of the solutions obtained above. The spin sum for the U-spinors is

$$\begin{aligned} P_U(y, y_\star, n, \vec{p}_{\bar{y}}) &\equiv \sum_s U_s(y, n, \vec{p}_{\bar{y}}) \overline{U}_s(y_\star, n, \vec{p}_{\bar{y}}) \\ &= \frac{1}{2(E_n + m)} \times \Big[\Big\{ m(1 + \sigma_z) + \not{p}_\parallel - \widetilde{\not{p}}_\parallel \gamma_5 \Big\} I_{n-1}(\xi) I_{n-1}(\xi_\star) \\ &\qquad + \Big\{ m(1 - \sigma_z) + \not{p}_\parallel + \widetilde{\not{p}}_\parallel \gamma_5 \Big\} I_n(\xi) I_n(\xi_\star) \\ &\qquad - \sqrt{2neB}(\gamma_1 - i\gamma_2) I_n(\xi) I_{n-1}(\xi_\star) \\ &\qquad - \sqrt{2neB}(\gamma_1 + i\gamma_2) I_{n-1}(\xi) I_n(\xi_\star) \Big] , \end{aligned} \tag{2.26}$$

where we have introduced the following notations for any object a carrying a Lorentz index:

$$\begin{aligned} a_\parallel^\mu &= (a_0, 0, 0, a_z) \,, \\ \widetilde{a}_\parallel^\mu &= (a_z, 0, 0, a_0) \,. \end{aligned} \tag{2.27}$$

Similarly, the spin sum for the V-spinors can also be calculated, and we obtain

$$\begin{aligned} P_V(y, y_\star, n, \vec{p}_{\bar{y}}) &\equiv \sum_s V_s(y, n, \vec{p}_{\bar{y}}) \overline{V}_s(y, n, \vec{p}_{\bar{y}}) \\ &= \frac{1}{2(E_n + m)} \times \Big[\Big\{ -m(1 + \sigma_z) + \not{p}_\parallel - \widetilde{\not{p}}_\parallel \gamma_5 \Big\} I_{n-1}(\widetilde{\xi}) I_{n-1}(\widetilde{\xi}_\star) \\ &\qquad + \Big\{ -m(1 - \sigma_z) + \not{p}_\parallel + \widetilde{\not{p}}_\parallel \gamma_5 \Big\} I_n(\widetilde{\xi}) I_n(\widetilde{\xi}_\star) \\ &\qquad + \sqrt{2neB}(\gamma_1 - i\gamma_2) I_n(\widetilde{\xi}) I_{n-1}(\widetilde{\xi}_\star) \\ &\qquad + \sqrt{2neB}(\gamma_1 + i\gamma_2) I_{n-1}(\widetilde{\xi}) I_n(\widetilde{\xi}_\star) \Big] . \end{aligned} \tag{2.28}$$

3 The fermion field operator

Since we have found the solutions to the Dirac equation, we can now use them to construct the fermion field operator in the second quantized version. For this, we write

$$\psi(X) = \sum_{s=\pm} \sum_{n=0}^{\infty} \int \frac{dp_x\, dp_z}{D} \Big[f_s(n, \vec{p}_{\not{y}}) e^{-ip\cdot X_{\not{y}}} U_s(y, n, \vec{p}_{\not{y}}) + \widehat{f}_s^\dagger(n, \vec{p}_{\not{y}}) e^{ip\cdot X_{\not{y}}} V_s(y, n, \vec{p}_{\not{y}}) \Big] \,. \quad (3.1)$$

Here, $f_s(n, \vec{p}_{\not{y}})$ is the annihilation operator for the fermion, and $\widehat{f}_s^\dagger(n, \vec{p}_{\not{y}})$ is the creation operator for the antifermion in the n-th Landau level with given values of p_x and p_z. The creation and annihilation operators satisfy the anticommutation relations

$$\Big[f_s(n, \vec{p}_{\not{y}}), f_{s'}^\dagger(n', \vec{p}\,'_{\not{y}}) \Big]_+ = \delta_{ss'} \delta_{nn'} \delta(p_x - p'_x) \delta(p_z - p'_z) \,, \quad (3.2)$$

and a similar one with the operators $\widehat{f}$ and $\widehat{f}^\dagger$, all other anticommutators being zero. The quantity D appearing in Eq. (3.1) depends on the normalization of the spinor solutions, and this is why of the spinors could have been chosen arbitrarily, as remarked after Eq. (2.17). Once we have chosen the spinor normalization, the factor D appearing in Eq. (3.1) is however fixed, and it can be determined from the equal time anticommutation relation

$$\Big[\psi(X), \psi^\dagger(X_\star) \Big]_+ = \delta^3(\vec{X} - \vec{X}_\star) \,. \quad (3.3)$$

Plugging in the expression given in Eq. (3.1) to the left side of this equation and using the anticommutation relations of Eq. (3.2), we obtain

$$\begin{aligned} \Big[\psi(X), \psi^\dagger(X_\star) \Big]_+ = \sum_s \sum_n \int \frac{dp_x\, dp_z}{D^2} \quad & \Big(e^{-ip_x(x-x_\star)} e^{-ip_z(z-z_\star)} U_s(y, n, \vec{p}_{\not{y}}) U_s^\dagger(y_\star, n, \vec{p}_{\not{y}}) \\ & + e^{ip_x(x-x_\star)} e^{ip_z(z-z_\star)} V_s(y, n, \vec{p}_{\not{y}}) V_s^\dagger(y_\star, n, \vec{p}_{\not{y}}) \Big) \,. \quad (3.4) \end{aligned}$$

Changing the signs of the dummy integration variables p_x and p_z in the second term, we can rewrite it as

$$\begin{aligned} \Big[\psi(X), \psi^\dagger(X_\star) \Big]_+ = \sum_s \sum_n \int \frac{dp_x\, dp_z}{D^2} \quad & e^{-ip_x(x-x_\star)} e^{-ip_z(z-z_\star)} \Big(U_s(y, n, \vec{p}_{\not{y}}) U_s^\dagger(y_\star, n, \vec{p}_{\not{y}}) \\ & + V_s(y, n, -\vec{p}_{\not{y}}) V_s^\dagger(y_\star, n, -\vec{p}_{\not{y}}) \Big) \,. \quad (3.5) \end{aligned}$$

Using now the solutions for the U and the V spinors from Eqs. (2.23) and (2.25), it is straight forward to verify that

$$\begin{aligned} & \sum_s \Big(U_s(y, n, \vec{p}_{\not{y}}) U_s^\dagger(y_\star, n, \vec{p}_{\not{y}}) + V_s(y, n, -\vec{p}_{\not{y}}) V_s^\dagger(y_\star, n, -\vec{p}_{\not{y}}) \Big) \\ = & \left(1 + \frac{p_z^2 + 2neB}{(E_n + m)^2} \right) \times \text{diag} \Big[I_{n-1}(\xi) I_{n-1}(\xi_\star), I_n(\xi) I_n(\xi_\star), I_{n-1}(\xi) I_{n-1}(\xi_\star), I_n(\xi) I_n(\xi_\star) \Big] \quad (3.6) \end{aligned}$$

where 'diag' indicates a diagonal matrix with the specified entries, and ξ and $\xi_\star$ involve the same value of p_x. At this stage, we can perform the sum over n in Eq. (3.5) using the completeness relation of Eq. (2.18), which gives the δ-function of the y-coordinate that should appear in the anticommutator. Finally, performing the integrations over p_x and p_z, we can recover the δ-functions for the other two coordinates as well, provided

$$\frac{2E_n}{E_n+m}\,\frac{1}{D^2}=\frac{1}{(2\pi)^2}\,, \tag{3.7}$$

using the expression for the energy eigenvalues from Eq. (2.19) to rewrite the prefactor appearing on the right side of Eq. (3.6). Putting the solution for D, we can rewrite Eq. (3.1) as

$$\begin{aligned}\psi(X) = & \sum_{s=\pm}\sum_{n=0}^{\infty}\int\frac{dp_x\,dp_z}{2\pi}\sqrt{\frac{E_n+m}{2E_n}} \\ & \times\Big[f_s(n,\vec{p}_{\not{y}})e^{-ip\cdot X_{\not{y}}}U_s(y,n,\vec{p}_{\not{y}})+\widehat{f}_s^{\dagger}(n,\vec{p}_{\not{y}})e^{ip\cdot X_{\not{y}}}V_s(y,n,\vec{p}_{\not{y}})\Big]\,. \end{aligned}\tag{3.8}$$

The one-fermion states are defined as

$$|n,\vec{p}_{\not{y}}\rangle = Cf^{\dagger}(n,\vec{p}_{\not{y}})\,|0\rangle\;. \tag{3.9}$$

The normalization constant C is determined by the condition that the one-particle states should be orthonormal. For this, we need to define the theory in a finite but large region whose dimensions are L_x, L_y and L_z along the three spatial axes. This gives

$$C=\frac{2\pi}{\sqrt{L_xL_z}}\,. \tag{3.10}$$

Then

$$\psi_U(X)\,|n,\vec{p}_{\not{y}}\rangle = \sqrt{\frac{E_n+m}{2E_nL_xL_z}}e^{-ip\cdot X_{\not{y}}}U_s(y,n,\vec{p}_{\not{y}})\,|0\rangle\;, \tag{3.11}$$

where ψ_U denotes the term in Eq. (3.8) that contains the U-spinors. Similarly,

$$\Big\langle n,\vec{p}_{\not{y}}\Big|\overline{\psi}_U(X) \;=\sqrt{\frac{E_n+m}{2E_nL_xL_z}}e^{ip\cdot X_{\not{y}}}\overline{U}_s(y,n,\vec{p}_{\not{y}})\,\langle 0|\;. \tag{3.12}$$

4 Inverse beta-decay

In this section, we calculate the cross section for the inverse beta-decay process $\nu_e+n\to p+e^-$ in a background magnetic field. The magnetic field might provide a net polarization of the neutrons, which we take into account. However, the magnitude of the field is assumed to be much smaller than m_n^2/e or m_p^2/e, so we ignore its effects on the proton and neutron spinors. The electron spinors, on the other hand, are the ones appropriate for the Landau levels. Thus, we can write the process as

$$\nu_e(\vec{k})+n(\vec{P})\to p(\vec{P}')+e(\vec{p}'_{\not{y}},n')\,. \tag{4.1}$$

4.1 The S-matrix element

The interaction Lagrangian for this process is

$$\mathscr{L}_{\rm int} = \sqrt{2}\, G_\beta \left[\overline{\psi}_{(e)}\gamma^\mu L\psi_{(\nu_e)}\right] \left[\overline{\psi}_{(p)}\gamma_\mu(g_V - g_A\gamma_5)\psi_{(n)}\right] , \tag{4.2}$$

where $L = \frac{1}{2}(1-\gamma_5)$ and $G_\beta = G_F \cos\theta_c$, θ_c being the Cabibbo angle. In first order perturbation, the S-matrix element between the final and the initial states of the process in Eq. (4.1) is therefore given by

$$\begin{aligned} S_{fi} = \sqrt{2}\, G_\beta \int d^4X \;\; & \left\langle e(\vec{p}_{\not y}', n') \left| \overline{\psi}_{(e)}\gamma^\mu L\psi_{(\nu_e)} \right| \nu_e(\vec{k}) \right\rangle \\ & \times \left\langle p(P') \left| \overline{\psi}_{(p)}\gamma_\mu(g_V - g_A\gamma_5)\psi_{(n)} \right| n(P) \right\rangle . \end{aligned} \tag{4.3}$$

For the hadronic part, we should use the usual solutions of the Dirac field which are normalized within a box of volume V, and this gives

$$\left\langle p(P') \left| \overline{\psi}_{(p)}\gamma_\mu(g_V - g_A\gamma_5)\psi_{(n)} \right| n(P) \right\rangle = \frac{e^{i(P'-P)\cdot X}}{\sqrt{2\mathcal{E}V}\sqrt{2\mathcal{E}'V}} \left[\overline{u}_{(p)}(\vec{P}')\gamma_\mu(g_V - g_A\gamma_5)u_{(n)}(\vec{P})\right] , \tag{4.4}$$

using the notations $\mathcal{E} = P_0$ and $\mathcal{E}' = P_0'$. For the leptonic part, we need to take into account the magnetic spinors for the electron. Using Eq. (3.12), we obtain

$$\left\langle e(\vec{p}_{\not y}', n') \left| \overline{\psi}_{(e)}\gamma^\mu L\psi_{(\nu_e)} \right| \nu_e(\vec{k}) \right\rangle = \frac{e^{-ik\cdot X + ip'\cdot X_{\not y}}}{\sqrt{2\omega V}} \sqrt{\frac{E_{n'} + m}{2E_{n'}L_xL_z}} \left[\overline{U}_{(e)}(y, n', \vec{p}_{\not y}')\gamma^\mu Lu_{(\nu_e)}(\vec{k})\right] . \tag{4.5}$$

Putting these back into Eq. (4.3) and performing the integrations over all co-ordinates except y, we obtain

$$S_{fi} = (2\pi)^3 \delta^3_{\not y}(P + k - P' - p') \left[\frac{E_{n'} + m}{2\omega V\; 2\mathcal{E}V\; 2\mathcal{E}'V 2E_{n'}L_xL_z}\right]^{1/2} \mathscr{M}_{fi} . \tag{4.6}$$

Here, $\delta^3_{\not y}$ implies, in accordance with the notation introduced earlier, the δ-function for all space-time co-ordinates except y. Contrary to the field-free case, we do not get 4-momentum conservation because the y-component of momentum is not a good quantum number in this problem. The quantity $\mathscr{M}_{fi}$ is the Feynman amplitude, given by

$$\mathscr{M}_{fi} = \sqrt{2}G_\beta \left[\overline{u}_{(p)}(\vec{P}')\gamma_\mu(g_V - g_A\gamma_5)u_{(n)}(\vec{P})\right] \int dy\; e^{iq_y y} \left[\overline{U}_{(e)}(y, n', \vec{p}_{\not y}')\gamma^\mu Lu_{(\nu_e)}(\vec{k})\right] , \tag{4.7}$$

using the shorthand

$$q_y = P_y + k_y - P_y' . \tag{4.8}$$

The transition rate in a large time T is given by $|S_{fi}|^2/T$. From Eq. (4.6), using the usual rules like

$$\begin{aligned} \left|\delta(\mathcal{E} + \omega - \mathcal{E}' - E_{n'})\right|^2 &= \frac{T}{2\pi}\, \delta(\mathcal{E} + \omega - \mathcal{E}' - E_{n'}) , \\ \left|\delta(P_x + k_x - P_x' - p_x')\right|^2 &= \frac{L_x}{2\pi}\, \delta(P_x + k_x - P_x' - p_x') , \\ \left|\delta(P_z + k_z - P_z' - p_z')\right|^2 &= \frac{L_z}{2\pi}\, \delta(P_z + k_z - P_z' - p_z') , \end{aligned} \tag{4.9}$$

we obtain

$$|S_{fi}|^2/T = \frac{1}{16}(2\pi)^3 \delta^3_{\not y}(P + k - P' - p') \frac{E_{n'} + m}{V^3\omega\mathcal{E}\mathcal{E}'E_{n'}} \left|\mathscr{M}_{fi}\right|^2 . \tag{4.10}$$

4.2 The scattering cross section

Using unit flux $1/V$ for the incident particle as usual, we can write the differential cross section as

$$d\sigma = V\, \frac{|S_{fi}|^2}{T} d\rho\,, \tag{4.11}$$

where $d\rho$, the differential phase space for final particles, is given in our case by

$$d\rho = \frac{L_x}{2\pi}\, dp'_x\, \frac{L_z}{2\pi}\, dp'_z\, \frac{V}{(2\pi)^3}\, d^3P'\,. \tag{4.12}$$

Therefore

$$\begin{aligned} d\sigma &= V\, \frac{|S_{fi}|^2}{T}\, \frac{L_xL_z}{(2\pi)^2}\, dp'_x\, dp'_z\, \frac{V}{(2\pi)^3}\, d^3P' \\ &= \frac{1}{64\pi^2} \delta^3_{\not y}(P+k-P'-p')\, \frac{E_{n'}+m}{\omega\mathcal{E}\mathcal{E}'E_{n'}}\, \Big|\mathscr{M}_{fi}\Big|^2 \frac{L_xL_z}{V}\, dp'_x\, dp'_z\, d^3P'\,. \end{aligned} \tag{4.13}$$

The square of the matrix element is

$$\Big|\mathscr{M}_{fi}\Big|^2 = 2G_\beta^2 \ell^{\mu\nu} H_{\mu\nu}\,, \tag{4.14}$$

where $H_{\mu\nu}$ is the hadronic part and $\ell^{\mu\nu}$ the leptonic part, whose calculation we outline now.

For the hadronic part, we can use the usual Dirac spinors because of our assumption that the magnetic field is much smaller than m_p^2/e. We will work in the rest frame of the neutron. Due to the presence of the background magnetic field, the neutrons may be totally or partially polarized. We define the quantity

$$S \equiv \frac{N_n^{(+)} - N_n^{(-)}}{N_n^{(+)} + N_n^{(-)}}\,, \tag{4.15}$$

where $N_n^{(\pm)}$ denote the number of neutrons parallel and antiparallel to the magnetic field. Then

$$H_{\mu\nu} = \frac{1}{2}(1+S)H_{\mu\nu}^{(+)} + \frac{1}{2}(1-S)H_{\mu\nu}^{(-)}\,, \tag{4.16}$$

where $H_{\mu\nu}^{(\pm)}$ denotes the contribution calculated with spin-up and spin-down neutrons respectively. Either of these contributions can be calculated by using the spin projection operator, which is $\frac{1}{2}(1\pm\gamma_5\gamma_3)$ for up and down spins. A straight forward calculation then yields

$$\begin{aligned} H_{\mu\nu} &= 2(g_V^2+g_A^2)(P_\mu P'_\nu + P_\nu P'_\mu - g_{\mu\nu}P\cdot P') \\ &\quad +2(g_V^2-g_A^2)m_nm_pg_{\mu\nu} + 4ig_Vg_A\varepsilon_{\mu\nu\lambda\rho}P^\lambda P'^\rho \\ &\quad - S\Big[4g_Vg_Am_n(P'_\mu g_{3\nu} + P'_\nu g_{3\mu} - P'_3 g_{\mu\nu}) + 2i\varepsilon_{\mu\nu3\alpha}R^\alpha\Big]\,, \end{aligned} \tag{4.17}$$

where we have introduced the shorthand

$$R^\alpha = (g_V^2+g_A^2)m_nP'^\alpha - (g_V^2-g_A^2)m_pP^\alpha\,. \tag{4.18}$$

We have omitted some terms in the expression for $H_{\mu\nu}$ that involve spatial components of the neutron momentum, with the anticipation that we will perform the calculation in the neutron rest frame.

In the leptonic part $\ell^{\mu\nu}$, we should use the magnetic spinors given in Sec. 2. This gives

$$\ell^{\mu\nu} = \int dy \int dy_\star \; e^{iq_y(y_\star - y)} \; \mathrm{Tr}\Big[P_U(y, y_\star, n', \vec{p}'_{\not{y}})\gamma^\mu \not{k} \gamma^\nu L\Big] , \tag{4.19}$$

where P_U denotes the spinor sum for the electrons, given in Eq. (2.26). We now have to perform the integrations over y and $y_\star$. Each of these variables should be integrated in the range $-\frac{1}{2}L_y$ to $+\frac{1}{2}L_y$. However, since we will take the infinite volume limit at the end as usual, we let $L_y \to \infty$ and use the result [10]

$$\int_{-\infty}^{+\infty} dy \; e^{ixy} I_n(y) = i^n \sqrt{2\pi} \; I_n(x) . \tag{4.20}$$

This gives

$$\ell^{\mu\nu} = \frac{2\pi}{eB} \frac{1}{(E_{n'} + m)} (\Lambda^\mu k^\nu + \Lambda^\nu k^\mu - k \cdot \Lambda g^{\mu\nu} - i\varepsilon^{\mu\nu\alpha\beta}\Lambda_\alpha k_\beta) , \tag{4.21}$$

where

$$\begin{aligned}\Lambda^\alpha = & \left[I_{n'-1}\left(\frac{q_y}{\sqrt{eB}}\right)\right]^2 (p'^\alpha_\parallel - \widetilde{p}'^\alpha_\parallel) + \left[I_{n'}\left(\frac{q_y}{\sqrt{eB}}\right)\right]^2 (p'^\alpha_\parallel + \widetilde{p}'^\alpha_\parallel) \\ & -2\sqrt{2n'eB} g_2^\alpha I_{n'}\left(\frac{q_y}{\sqrt{eB}}\right) I_{n'-1}\left(\frac{q_y}{\sqrt{eB}}\right) .\end{aligned} \tag{4.22}$$

Thus,

$$\begin{aligned}\Big|\mathscr{M}_{fi}\Big|^2 = & \; 8G_\beta^2 \times \frac{2\pi}{eB} \frac{1}{(E_{n'} + m)} \bigg[(g_V^2 + g_A^2)(P \cdot \Lambda \; P' \cdot k + P' \cdot \Lambda \; P \cdot k) \\ & -(g_V^2 - g_A^2) m_n m_p k \cdot \Lambda - 2g_V g_A (P \cdot \Lambda \; P' \cdot k - P' \cdot \Lambda \; P \cdot k) \\ & +S\Big(2g_V g_A m_n (P' \cdot \Lambda k_z + P' \cdot k \Lambda_z) - \Lambda_z k \cdot R + k_z \Lambda \cdot R\Big)\bigg] .\end{aligned} \tag{4.23}$$

We now choose the axes such that the 3-momentum of the incoming neutrino is in the x-z plane. We will also assume that $|\vec{P}'| \ll m_p$ for the range of energies of interest to us. In that case, it is easy to see that the terms involving $\sqrt{2n'eB}$ drop out, and we obtain

$$\begin{aligned}\Big|\mathscr{M}_{fi}\Big|^2 = 8G_\beta^2 \times \frac{2\pi}{eB} \frac{m_n m_p}{E_{n'} + m} \times & \Big[(g_V^2 + 3g_A^2)\omega\Lambda_0 + (g_V^2 - g_A^2) k_z \Lambda_z \\ & +2g_A S\Big((g_V - g_A)\omega\Lambda_z + (g_V + g_A) k_z \Lambda_0\Big)\Big] .\end{aligned} \tag{4.24}$$

We now put this expression into Eq. (4.13) and calculate the total cross section by performing the integrations over different final state momenta appearing in that formula. First we integrate over P'_x and P'_z. These appear only in the momentum conserving δ-function. Integration over them therefore just gets rid of the corresponding δ-functions. For the integration over p'_x, we refer to Eq. (2.12). Since the center of the oscillator has to lie between $-\frac{1}{2}L_y$ and $\frac{1}{2}L_y$, we conclude that $-\frac{1}{2}L_y eB \le p'_x \le \frac{1}{2}L_y eB$. Thus the integration over p'_x gives a factor $L_y eB$.

Putting back into Eq. (4.13) and using $V = L_x L_y L_z$, we obtain

$$d\sigma = \frac{G_\beta^2}{4\pi} \frac{\delta(Q+\omega-E_{n'})}{\omega E_{n'}} \times \Big[(g_V^2+3g_A^2)\omega\Lambda_0 + (g_V^2-g_A^2)k_z\Lambda_z \\ +2g_A S\Big((g_V-g_A)\omega\Lambda_z + (g_V+g_A)k_z\Lambda_0\Big)\Big]\, dP'_y dp'_z\,, \quad (4.25)$$

where Q is the neutron-proton mass difference, $m_n - m_p$.

We next perform the integration over P'_y. In the integrand, it occurs only as the argument of the functions I_n and I_{n-1}. The functions I_n are orthogonal in the sense that

$$\int_{-\infty}^{+\infty} da\; I_n(a) I_{n'}(a) = \sqrt{eB}\;\delta_{nn'}\,. \quad (4.26)$$

This property can be used to perform the integration over P'_y. We have already remarked that the term proportional to $\sqrt{2n'eB}$ in Eq. (4.22) does not contribute. From other two terms, we obtain

$$\begin{aligned} \int dP'_y\,\Lambda^\alpha &= eB\Big[(p'^\alpha_\parallel - \widetilde{p}'^\alpha_\parallel)(1-\delta_{n',0}) + (p'^\alpha_\parallel + \widetilde{p}'^\alpha_\parallel)\Big] \\ &= eB\Big[g_{n'} p'^\alpha_\parallel + \delta_{n',0}\widetilde{p}'^\alpha_\parallel\Big]\,, \end{aligned} \quad (4.27)$$

where

$$g_{n'} = 2 - \delta_{n',0} \quad (4.28)$$

gives the degeneracy of the Landau level. Notice the appearance of the Kronecker delta, $\delta_{n',0}$, in the expression of Eq. (4.27). The reason for this is that, while two terms of Eq. (4.22) contribute in the integral for $n' \neq 0$, only one of them contributes for $n' = 0$ since $I_{-1} = 0$.

The final integration is over p'_z. Writing the argument of the remaining δ-function in terms of p'_z, we find that the zeros occur when

$$p'_z = p'_\pm \equiv \pm\sqrt{(Q+\omega)^2 - m^2 - 2n'eB}\,. \quad (4.29)$$

Therefore,

$$\delta(Q+\omega-E_{n'}) = \frac{Q+\omega}{\sqrt{(Q+\omega)^2 - m^2 - 2n'eB}}\Big(\delta(p'_z - p'_+) + \delta(p'_z - p'_-)\Big)\,. \quad (4.30)$$

In the integration, the terms proportional to p'_z in the integrand receive equal and opposite contributions from the two δ functions and cancel. For the other terms, independent of p'_z, both the contributions are equal. So we obtain

$$\sigma_{n'} = \frac{eBG_\beta^2}{2\pi} \Big[g_{n'}\Big\{(g_V^2+3g_A^2) + 2g_A S(g_V+g_A)\cos\theta\Big\} \\ +\delta_{n',0}\Big\{(g_V^2-g_A^2)\cos\theta + 2g_A S(g_V-g_A)\Big\}\Big] \frac{Q+\omega}{\sqrt{(Q+\omega)^2 - m^2 - 2n'eB}}\,, \quad (4.31)$$

where we have defined the direction of the incoming neutrino by the angle θ, with

$$k_z = \omega\cos\theta\,. \quad (4.32)$$

In Eq. (4.31), we have denoted the cross section by $\sigma_{n'}$ because the electron ends up in a specific Landau level n'. The total cross section is then given as a sum over all possible values of n', i.e.,

$$\sigma = \sum_{n'=0}^{n'_{\rm max}} \sigma_{n'} = \frac{eBG_\beta^2}{2\pi} \sum_{n'=0}^{n'_{\rm max}} \Big[g_{n'} \Big\{ (g_V^2 + 3g_A^2) + 2g_A S(g_V + g_A)\cos\theta \Big\} + \delta_{n',0} \Big\{ (g_V^2 - g_A^2)\cos\theta + 2g_A S(g_V - g_A) \Big\} \Big] \frac{Q+\omega}{\sqrt{(Q+\omega)^2 - m^2 - 2n'eB}} . \quad (4.33)$$

The possible allowed Landau level has a maximum, $n'_{\rm max}$, which is given by the fact that the quantity under the square root sign in the denominator of Eq. (4.33) must be non-negative, i.e.,

$$n'_{\rm max} = {\rm int} \left\{ \frac{1}{2eB} \Big[(Q+\omega)^2 - m^2 \Big] \right\} . \quad (4.34)$$

Eq. (4.33) gives our result for the cross section of the inverse beta decay process. Some properties of this formula are worth noting.

For unpolarized neutrons, $S = 0$, the cross section for $n' \neq 0$ does not depend on the direction of the incoming neutrino. The same is not true if the electron ends up in the lowest Landau level. The cross section will be asymmetric in this case.

All terms in the cross section which depend on S have a common factor g_A. The reason is that, if g_A were equal to zero, the interaction in the hadronic sector would have been spin-independent.

If the final electron is in the lowest Landau level and the initial neutrino momentum is antiparallel to the magnetic field, Eq. (4.33) shows that

$$\sigma_0 = \frac{eBG_\beta^2}{2\pi} \Big[4g_A^2(1-S) \Big] \frac{Q+\omega}{\sqrt{(Q+\omega)^2 - m^2}} . \quad (4.35)$$

Note that the vector coupling g_V does not contribute to the cross section in this limit. This can be understood easily. The neutrino spin is along the $+z$ direction whereas the electron spin in the lowest Landau level must be in the $-z$ direction. Thus there is a spin-flip in the leptonic sector. Conservation of angular momentum then implies that there must be a spin-flip in the hadronic sector as well. In the non-relativistic limit for hadrons that we have employed, this can occur only through the axial coupling.

If further we consider totally polarized neutrons, i.e., $S = 1$, we see that σ_0 vanishes. Again, this is a direct consequence of angular momentum conservation. Since both initial particles have spin up, angular momentum conservation requires both final particles in spin up states as well. But the spin-up state is not available for the electron in the lowest Landau level.

It is instructive to check that the result obtained in Eq. (4.33) reduces to the known result for the field-free case. The contribution specific to the zeroth Landau level vanishes in the limit $B \to 0$ owing to the overall factor of eB. The other terms also have the factor eB, but in this case we also need to sum over infinitely many states. This gives

$$\sigma = \frac{eBG_\beta^2}{\pi} \Big[(g_V^2 + 3g_A^2) + 2g_A S(g_V + g_A)\cos\theta \Big] \times \left(\sum_{n'=0}^{n'_{\rm max}} \frac{Q+\omega}{\sqrt{(Q+\omega)^2 - m^2 - 2n'eB}} - \frac{Q+\omega}{2\sqrt{(Q+\omega)^2 - m^2}} \right) . \quad (4.36)$$

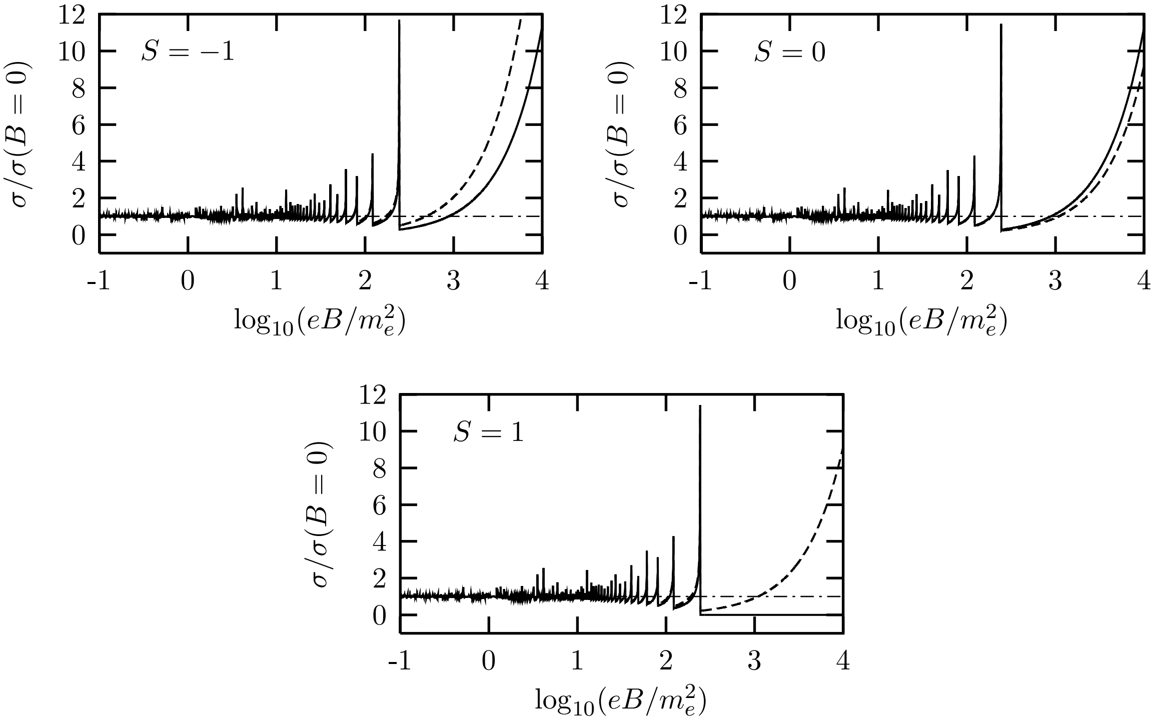

Figure 1: Enhancement of cross section in a magnetic field for an initial neutrino energy of 10 MeV. Different panels show the results for different net polarizations of the neutrons. The solid and the dashed lines correspond to the initial neutrino momentum parallel and antiparallel to the magnetic field. Each curve has been normalized by the cross section in field-free case for the same values of S and $\cos\theta$. The horizontal dashed lines represent unity in the vertical scale.

For $B \to 0$, the last term vanishes, and we can identify $n'_{\rm max}$ as the integer for which the denominator of the summand vanishes. Thus we obtain

$$\begin{aligned} \sigma &\longrightarrow \frac{eBG_\beta^2}{\pi}\Big[(g_V^2+3g_A^2)+2g_A S(g_V+g_A)\cos\theta\Big]\int_0^{n'_{\rm max}} dn' \; \frac{Q+\omega}{\sqrt{(Q+\omega)^2-m^2-2n'eB}} \\ &= \frac{G_\beta^2}{\pi}\Big[(g_V^2+3g_A^2)+2g_A S(g_V+g_A)\cos\theta\Big](Q+\omega)\sqrt{(Q+\omega)^2-m^2}\,, \end{aligned} \tag{4.37}$$

which is the correct result in the field-free case.

In Fig. 1, we have plotted the ratio of the cross section to its corresponding value at $B=0$ as a function of the magnetic field. The plots have been done for unpolarized ($S=0$) as well as totally polarized neutrons along ($S=1$) and opposite ($S=-1$) to the magnetic field, with the initial neutrino momentum parallel and antiparallel to the magnetic field. For $S=-1$, we find that neutrinos parallel to the magnetic field have a smaller cross section than those antiparallel to the field, and the difference is pronounced for large fields. For $S=0$, the situation is just reversed. For $S=1$, if the magnetic field is high enough so that $n'_{\rm max}=0$, we see that the cross section vanishes for neutrino momentum antiparallel to the field. The reason for this has already been discussed.

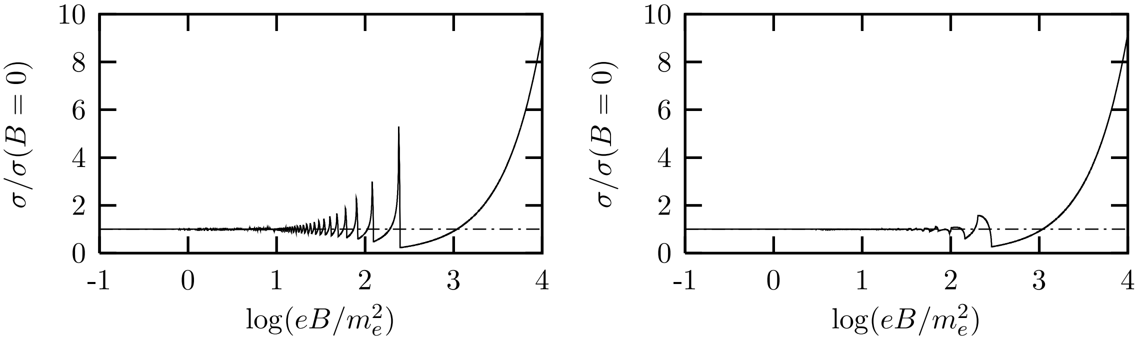

Figure 2: Cross section for unpolarized neutrons as a function of the magnetic field for a flat energy distribution, normalized to the cross section in the field-free case. The initial neutrino momentum is along the magnetic field, and energy is 10 MeV. The energy spread $\omega_2 - \omega_1$ is 0.2 MeV for the left panel and 2 MeV for the right panel.

5 Consequences of neutrino energy spread

The enhancement factor in Fig. 1 shows some spikes. They appear at values of the magnetic field for which the denominator of Eq. (4.33) vanishes for some n'. For field values larger than this, that particular Landau level does not contribute to the cross section. To the right of the final spike that appears in the figure, only the zeroth Landau level contributes. In other words, the final electron can go only to the lowest Landau level for such high values of the magnetic field. The exact value of B for which this occurs depends of course on the energy of the initial neutrino.

We need to make an important point about these spikes. Each spike in fact go all the way up to infinity. The finite height of a spike in the figure is an artifact of the finite step size taken in plotting it.

In reality, of course, a cross section cannot be infinite. In the present case, this is related to the fact that the initial neutrinos cannot be exactly monochromatic due to the uncertainty relation. There must be a spread in energy, which can be represented by a probability distribution $\Phi(\omega)$, defined by

$$\int d\omega \ \Phi(\omega) = 1 \,. \tag{5.1}$$

In that case, the cross section in a real experiment should be written in the form

$$\sigma = \int d\omega \ \Phi(\omega)\sigma(\omega) \,, \tag{5.2}$$

where $\sigma(\omega)$ is the expression derived in Eq. (4.33) for a single value of energy.

As an illustration, we consider the case of unpolarized neutrons ($S = 0$), and take a flat probability distribution of initial neutrino energy, viz.,

$$\Phi(\omega) = \begin{cases} \dfrac{1}{\omega_2 - \omega_1} & \text{if } \omega_1 \leq \omega \leq \omega_2, \\ 0 & \text{otherwise.} \end{cases} \tag{5.3}$$

Then the integration of Eq. (5.2) gives

$$\sigma = \frac{eBG_\beta^2}{2\pi(\omega_2 - \omega_1)} \Big[F(\omega_2) - F(\omega_1)\Big] , \tag{5.4}$$

where

$$F(\omega) = \sum_{n'=0}^{n'_{\rm max}} \left[g_{n'}(g_V^2 + 3g_A^2) + \delta_{n',0}(g_V^2 - g_A^2)\cos\theta \right] \times \sqrt{(Q+\omega)^2 - m^2 - 2n'eB} \,, \tag{5.5}$$

with $n'_{\rm max}$ determined by Eq. (4.34). Fig. 2 shows the variation of this quantity with the magnetic field for $\cos\theta = 1$. In this figure, we normalize the cross section by $B = 0$ cross section with the energy distribution of Eq. (5.3), which is

$$\sigma(B=0) = \frac{G_\beta^2(g_V^2 + 3g_A^2)}{3\pi(\omega_2 - \omega_1)} \left(\Big[(Q+\omega_2)^2 - m^2\Big]^{3/2} - \Big[(Q+\omega_1)^2 - m^2\Big]^{3/2} \right) . \tag{5.6}$$

Keeping the central value of neutrino energy as 10 MeV as before, we have drawn these plots for two different values of the spread, as mentioned in the caption. For the smaller value of the spread in particular, the graph looks very similar to that drawn in Fig. 1, but the difference is that now the height of the spikes denote the actual enhancement, and is not an artifact of the plotting procedure. For the higher value of the energy spread, we see that the spikes have smoothened out.

6 Comments

The calculation of the cross section for inverse beta decay process has been performed earlier by several authors [6, 7]. They assumed that the matrix element remains unaffected by the magnetic field, only the modified phase space integral makes the difference in the cross section. The results they obtained is the same as the term proportional to $g_V^2 + 3g_A^2$ that we obtained.

We have used exact spinor solutions in a uniform magnetic field to calculate the cross section of the inverse beta decay process for arbitrary neutron polarization. We find that, even for unpolarized neutrons, the cross section depends on the direction of the neutrino momentum. This asymmetry is not surprising since the background magnetic field makes it an anisotropic problem. A similar asymmetry has been noted for URCA processes [5], where the neutrino is in the final state. The anisotropy for the $S = 0$ case comes only from the $n' = 0$ contribution [3]. However, for $n' \neq 0$, there is a cancellation between the two possible states in a Landau level which washes out all angular dependence in these levels, provided the neutrons are unpolarized. The asymmetry in cross section will therefore come only from the $n' = 0$ state and its amount will depend on the relative contribution of this state to the total cross section. If the magnetic field is so high that only the $n' = 0$ state can be obtained for the electron, the asymmetry will be large, about 18%. For smaller and smaller magnetic fields, the asymmetry decreases with new Landau levels contributing. For polarized neutrons, however, there is an asymmetry even in the field-free case. In presence of a magnetic field, the asymmetry will in general depend on the magnetic field, as it appears from various plots in Fig. 1.

This fact can have far reaching consequences for neutrino emission from a proto-neutron star. It has been discussed in the literature that the presence of asymmetric magnetic fields can cause asymmetric neutrino emission from a proto-neutron star [11]. However, our calculations show that even with a uniform magnetic field, neutrino emission would be asymmetric because of the $\cos\theta$-dependent terms in the cross section.

Acknowledgements : We thank Debades Bandopadhyay, A. Y. Parkhomenko and Esteban Roulet for illuminating discussions and for pointing out some important papers in the field. We also thank Duane Dicus and Huaiyu Duan for pointing out some mistakes in an earlier version of our work.

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